A

Synopsis

On

"Panorama Behavior of the Physical Distributions in Modified Theories of Gravitation"

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1. Introduction:

At the start of the twentieth century, Albert Einstein proposed the General Theory of Relativity, which modifications our knowledge of the universe. It’s miles growing with the aid of a outstanding range of correct observation and exploring the hidden eventualities of the universe in modern - day. Later on, a group of supernovae observations confirms that our universe is currently going through the accelerated expansion phase. This cause the existence of high negative pressure in the universe which produce by the unknown form of energy and matter called dark energy (DE) and dark matter (DM). In the last two decades many works are carried out in modified gravity theories to explore and understand the profile of the universe.

A cosmological model provides a description of the dynamic and large-scale structures of the universe and allows study of fundamental questions about its origin, structure, evolution and ultimate fate. Realistic cosmological model usually has the energy momentum tensor of matter generated by a perfect fluid. The matter distribution is satisfactorily described by perfect fluids in our universe. A realistic treatment of the problem requires the consideration of material distribution other than perfect fluid. As we know according to Big Bang cosmology, when neutrino decompiling occurs, the matter behaves as viscous fluid cosmological model of early universe has been widely discussed in the literature.

GR provides a unified description of gravity as a geometric property of space and time. Curvature of space-time is directly related to the four-momentum of whatever matter and radiation are present. GR has developed into an essential tool in modern astrophysics. It provides the foundation of understanding of black holes, regions of space where gravitational attraction is so strong that even light can’t escape. As Albert Einstein later said, the reason for development of GR was that the preference of inertial motions within Special Relativity (SR) was unsatisfactory, while a theory which from the outset prefers no state of motion (even accelerated ones) should appear more satisfactory.

Einstein theory of gravitation was very much successful for constructing a cosmological model and explaining the origin and evolution of the universe. But Einstein theory fails to explain late time acceleration of modern cosmology. So many attempts have been made to modify the gravity theory which is to gratify the present

accelerated phase. Many modified gravity theories have been proposed to describe the late and early acceleration of the universe. The $f(R)$ gravity proposed in Buchdahl (1970), is the most fundamental and widely used modification to GR. A natural generalization is to choose a more general action in which the standard Einstein-Hilbert action is replaced by an arbitrary function of the Ricci scalar R (Nojiri and Odintsov 2003b) (i.e, $f(R)$) and is the name applied to $f(R)$-gravity. This modified theory may point this late time cosmic acceleration (Carroll *et al.* 2004). These $f(R)$ models can satisfy local tests and integrate inflation with dark energy. For a brief review of $f(R)$-gravity one may refer to Nojiri and Odintsov (2003b), Carroll *et al.* (2004) and Capozziello *et al.* (2003). This modified gravity has recently been verified to explain the late time accelerated expansion of the Universe. One of the interesting and prospective versions of modified gravity theory is the $f(R,T)$ gravity proposed by Harko *et al*. (2011) where, the gravitational Lagrangian is given by n arbitrary function of the Ricci scalar R and the trace of the stress-energy tensor T. They have obtained the gravitational field equations in the metric formalism as well as the equation of motion for test particles, which follows the covariant divergence of the stress-energy tensor. Akta¸ s and Aygün (2017) have also discussed Magnetized strange quark matter solutions in $f(R,T)$ gravity with a cosmological constant and it proves that the $f(R,T)$ gravity models can explain the late time cosmic accelerated expansion of the Universe.

An alternative approach to accommodate the current accelerating expansion of the universe is to modify the GR on large scale, such as, the scalar-tensor theories, $f(R)$ theory, $f(T)$ theory and DGP braneword theory (Yang 2011a; Zheng and Huang 2011) etc. Among these theories, the generalized teleparallel theory of gravity has recently gained a lot of interest due to its possible explanation about DE. In this generalization, the torsion scalar, T, is replaced by its general function $f(T)$ in the Lagrangian of teleprallel gravity (Sharif and Rani 2011a). The $f(T)$ gravity models use the Weitzenb¨ock connection which inherits only torsion and responsible for the accelerating expansion of the universe (Myrzakulov 2011). This approach (theory) originally developed by Einstein in 1928 under the name ”Fern-Parallelismus” or ”distant Parallelism” or ”teleparallelism” (Linder 2010; Unzicker and Case 2005). The $f(T)$ theory has significant advantage of its second order field equations as compared to $f(R)$ theory with fourth order field equations (Sharif and Shamir 2009; Sharif and Kausar 2010, 2011a, 2011b; Jamil et al. 2012a, 2012b).

Bengochea and Ferraro (2009) described the recently detected acceleration of the universe without DE and performed observational viability tests by using recent SNIa data for some $f(T)$ models. Linder (2010) explored that the cosmological constant is not only the component for the acceleration of the universe. This was also observed through a generalization of GR to other gravity theories. Zhang et al. (2011)

discussed the dynamical analysis for the logarithmic and power form of $f(T)$ models. They investigated phase space analysis of models, their stability of the critical points and evolution of 2 EoS parameter. In a recent paper, Sharif and Shamaila (2011b) constructed some $f(T)$ models by using Bianchi type I (BI) universe. They also derived EoS parameter for two modified teleparallel models. Yang (2011b) investigated three new $f(T)$ models and described their physical implications and cosmological behaviour. With the help of available observational data, some physical characteristics will be studied through the cosmological parameters, which are in good agreement with the observationally obtained present facts of the universe.

Similarly, the exist several other modified theories with different cosmological implications such as $f(G)$ theory (De Felice and Tsujikawa,2009), $f(R,G)$ theory (Elizalde et al.,2010), $f(T,B)$ theory (Bahamonde et al., 2018) etc. Further, Wely proposed an extension of Riemannian geometry, in which he established the first unified theory of gravity and electromagnetism, where the non- metricity of spacetime generated the electromagnetic field. As a result, the symmetric teleparallel representation is the third generalization of GR. This generalization is extended to the non-metric gravity known as the $f(Q)$ gravity (Jimenez et al.,2018, 2020), where Q is the non - metricity representing the geometric variable defining the characteristics of the gravitational interaction. Recently, Xu et al.,(2019, 2020) have proposed a new extension of $f(Q)$ known as the $f(Q,T)$ theory where the non - metricity Q is non-minimally coupled to the trace T of the energy- momentum tensor.

2. Review of Literature and Development in the Subject:

One of the predictions from inflation is that the observed universe should nearly isotropic on large scales. Spatially homogenous and isotropic universe can be well described by FRW model which cannot be explained the early universe. The existence of anisotropy at early times is a natural phenomenon to investigate the local anisotropies that we observe today in galaxies and cluster. Recent measurements on the cosmic microwave background radiation (CMBR) through WMAP suggest that the anisotropic inflationary models should be considered (Vielva 2004; Costa 2004). In early epoch of the big bang, the universe does not maintain its isotropic behavior at very small scales (Kumar and Singh 2008). In order to gets a realistic model representing an expanding, homogenous and anisotropic universe, Bianchi type cosmological models are considered. The isotropic behaviour of today universe makes BI model a prime candidate for describing the possible anisotropic effects of the early universe on modern day data observations.

Many people worked on simplest anisotropic model called BI universe to discuss the effects of anisotropy in several contexts. Kumar and Singh (2007) developed mechanism for solution of the field equations by considering BI universe

model. The same authors (Kumar and Singh 2008) investigated the exact BI solutions which give the constant value of the deceleration parameter in scalar-tensor theory. Recently, Sharif and Waheed (2012) studied exact solutions for anisotropic fluids by taking the locally rotationally symmetric Bianchi Type I (LRS BI) universe model which generalizes the flat FRW universe in the modified theory (BD). There are many papers available in literature where this model has been used widely (Bali and Kumawat 2008; Amirhashchi 2011; Yadav and Saha 2012). Recently, Bamba et al. (2011) discussed different $f(T)$ models to investigate the cosmological evolutions of EoS parameter for DE and their observational constraints. Wu and Yu (2011) proposed two new $f(T)$ models and analyzed that the crossing of the phantom divide line is consistent with recent cosmological observational data. Chen et al. (2011) investigated this problem by extending modified gravity at the background and perturbed level. They also explored this theory for quintessence scenario.

Recently, Samanta and Myrzakulov (2017) have studied imperfect fluid cosmological model in $f(R,T)$ gravity, Samanta et al. (2017) have studied Kaluza Klein bulk viscous fluid cosmological models and the validity of the second law of thermodynamics in $f(R,T)$ gravity. Also Keskin (2018), Sharif (2018), Chaubey and Shukla (2013), Samanta (2013a, b), and Reddy et al. (2012, 2013) have studied cosmological models in $f(R,T)$ gravity in different Bianchi type space-times. Presently, Bianchi universes are playing important role in observational cosmology, since the WMAP data (Nojiri et al. 2008; Bamba et al. 2012) seem to require an addition to the standard cosmological model with positive cosmological constant that bears a likeness to the Bianchi morphology (Jamil et al. 2012a, b; Steigman & Turner 1983; Hinshaw et al. 2009). Katore et al. (2015) have discussed domain walls in $f(R,T)$ theory. Agrawal and Pawar (2017a, b) have studied Bianchi type-V universe model with magnetized domain walls in $f(R,T)$ theory. Also, Agrawal and Pawar (2017a, b) obtained plane symmetric cosmological model with quark and strange quark matter in $f(R,T)$ theory of gravity.T Harko and et al., (2018) have studied coupling matter in modified Q gravity; S. Capozziello (2019) have studied extended gravity cosmography; Yixin Xu.et al.,(2019) have studied $f(Q,T)$ gravity; Z.Yousaf and M.Z. Batti and T. Naseer (2020) have studied study of static charged sperical structure in $f(R,T,Q)$ gravity; S.H. Shekh (2021) have studied models of holographic dark energy in f(Q) gravity; Simran Arora, JRL Santos and P K Sahoo (2021) have studied constraining $f(Q,T)$ gravity from energy conditions; Pati, Laxmipriya, B. Mishra, and S. K. Tripathy (2021) have studied "Model parameters in the context of late time cosmic acceleration in $f(Q,T)$ gravity." M.Koussour and Pacif et al., (2022) have studied Late-time acceleration in $f(Q)$ gravity: Analysis and constraints in an anisotropic background; Sanjay Mandal, Sahoo and JRL Santos (2022) have studied comment on "Energy conditions in $f(Q)$ gravity "; Pawar, D. D., D. K. Raut, and W. D. Patil (2022) have studied "FRW model with two-fluid source in fractal

cosmology." Avik De et al. (2022) have studied isotropization of locally rotationally symmetric Bianchi - I universe in $f(Q)$ gravity. The Bianchi-V locally rotationally symmetric cosmologies (LRS type-V) have commanded the attention of many researchers during the past decade. The reason is that, while their dynamical behavior can be far more general than FLRW and Bianchi-I model, they are still less complicated than Bianchi type II, VIII and IX.

To understand the dark energy problem in a better way, time-to-time various cosmologists have proposed some well-known modified theories of gravitation viz. $f(G)$ gravity, $f(R)$ gravity, $f(R,T)$ gravity, $f(G,T)$ gravity, $f(R,T,R_{\mu\nu}T^{\mu\nu})$ gravity, $f(Q)$ theory, $f(Q,T)$ theory and recently fractal gravity . $f(R)$ is a type of modified gravity theory which generalizes Einstein's general relativity.

Mostly the expansion of the universe is described within the framework of the homogeneous and isotropic Friedman–Robertson–Walker (FRW) cosmology. The reasons for this are purely technical. The simplicity of the field equations and the existence of analytical solutions in most of the cases have justified this over simplification for the geometry of space-time. However, there are no compelling physical reasons to assume the former before the inflationary period. To drop the assumption of homogeneity would make the problem intractable, while the isotropy of the space is something that can be relaxed and leads to anisotropy. Nowdays we used 57 points of the Hubble data points and 1048 pantheon data points to constrain the model parameters.

3. Aim and Objectives of Research:

Considering different types of space-times including Bianchi types, Plane symmetric, Kantowski-Sachs FRW space time and Kasner space-times, we will obtain the exact solutions of the field equations, with perfect fluid, dark energy and dark matter in alternative theories of gravitation like Scalar-tensor theory, Brans-Dicke theory, Barber's self creation theory, Saez –Ballester's theory, $f(R)$, $f(R,T)$, $f(T)$, $f(Q,T)$, $f(Q)$, $f(T,B)$, $f(R,G)$ theory of gravity, etc . Some physical and geometrical properties of the models will be studied.

4. Methodology:

Knowledge of the concern topic will be obtained by the survey of research papers, review articles, various books and journals. Also we would like to visit different research institutes like IIT's, TIFR, IUCAA Pune, BHU Varanasi, IISC, SGBA Uni. Amravati, RTM Uni. Nagpur, GLA University Mathura, Jadavpur University, Kolkata etc to interact with eminent Mathematicians and Physicist also would like to attend national, international conference related to our research work.

5. Importance of Study:

Basic aim of our study is to determine the large-scale structure and evolution of the universe which seems to be dominated by the mysterious form of energy, called dark energy. Another basic role of study is to obtain the required cosmological model and to investigate it by comparing with present day universe using astronomical data. A true cosmological constant can act as dark energy. However, this constant has a difficult theoretical interpretation and suffers the fine-tuning problem. Besides it gives no solution for the coincidence problem. However, do we need a fundamental solution for dark energy dominance now? Assuming that the coincidence problem has a solution, a dynamical mechanism for dark energy is a good candidate. Since it seems that about 70% of the energy of the universe is not under-stood, the nature of dark energy is one of the most exciting and fundamental question of physics today. The main purpose of the present work is to study the dynamical and physical behaviour of dark energy in the evolution of universe; it will be useful for young researchers.

6. Plan of Research Work:

Sr.No	Period	Plan of the work	Targets to be achieved
1	First Year	Review of Research Papers, Collecting the Papers.	Good command on the subject, clear the course work and try to get some construction techniques.
2	Second Year	Analyze Papers & Work on some problems to be included in the thesis.	Publish at least 2 or 3 papers.
3	Third Year	Writing final thesis and submitting the thesis.	Work on some more papers and thesis submission.

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